Study on the calculation method for hydrodynamic pressure of bridge piers in deep water under earthquakes

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China has a planning of five sea-crossing projects: the Bohai Strait, the Yangtze River Estuary, Hangzhou Bay, Pearl River Estuary and the Qiongzhou strait from north to south.

Typical deep water bridges in domestic

Yangtze River Bridge in Yunyang

Total length: 639m
Type: Cable-stayed Bridge
Max water depth: 96m
1. The Background of Research

The damage of Bridge in 2008 Wenchuan earthquake: The main pier of the Miaoziping bridge is 104 m high and its deepest submerged depth is more than 40 m. Although there is no evidence that the impact of hydrodynamic pressure could directly result in structural damage, the influence of water on structures should nevertheless be considered in order to ensure the safety of structures in water.

FIGURE 1
The damages of Mingjiang Bridge in Miaoziping (color figure available online).
The Application requirements of study

Taking the performance design of steel arch bridge for example: the structural design of the pier and abutment, including the main beam are closely related to hydrodynamic effect.

Application: Bending Moment of Girder and Pier > Energy transfer
1. The Background of Research

In this study, based on the ideal potential fluid theory and the Morison equation, a simplified calculation method is put forward. The dynamic response of a typical cylindrical pile was studied by using the simplified method and then the results of the model were compared with those from the finite element method. In order to further validate the simplified method, a comparison of results from simplified method and the experiment was performed.
1. The Background of Research

Seismic action - water bridge structure

Earthquake damage investigation

Experimental study

Theoretical analysis

Field test

Laboratory test

Numerical method

Simplified method

Seismic observation

Shaking table test

Explain test and seismic damage

Meets the demand of engineering seismic design method

Research Frame
2. The Effect of Hydrodynamic Pressure on the Slender Structure Based on the Simplified Method

The distribution of hydrodynamic pressure long the height of piers under the earthquake

It can be found that hydrodynamic pressure caused by plate boundary type seismic wave acting on the piers is larger than large-scale earthquake occurred at the inland area and the obvious peak value of hydrodynamic pressure appeared on the top of the pier when the plate boundary type occurred. This will make the point of hydrodynamic pressure getting upper and bigger bending moment at the bottom of the piers.
3. The Effect of Hydrodynamic Pressure on the Slender Structure Based on the Simplified Method

For typical entity rectangular section and cylindrical pile pier structure, studying on the distribution of hydrodynamic pressure acting on the structure. Analyzing the influence factors of dynamic water - structure interaction, hydrodynamic effect on dynamic characteristics of the natural vibration period of piers cannot be ignored when water depth value exceeding the critical depth.

T1-II-3 different water depths under seismic action hydrodynamic pressure distribution along the pier height direction

(a) 20m depth of water  (b) 15m depth of water  (c) 10m depth of water  (d) 5m depth of water

T2-II-1 different water depths under seismic action hydrodynamic pressure distribution along the pier height direction

(a) 20m depth of water  (b) 15m depth of water  (c) 10m depth of water  (d) 5m depth of water
2. A Simplified Calculation Method for Hydrodynamic Pressure to Slender Structures

According to Morison’s hydrodynamic theory, the pier in the water will produce the same magnitude of movement as that of the fluid particles. The hydrodynamic force on the pile includes two components: one is when the fluid flows through the piles, the existence of the piles causes the acceleration or deceleration of the fluid, and the fluid exerts a force on the pile known as the inertial force $F_I$; the other is due to the viscous effect and the swirl in the wake of the fluid which causes a drag force on the pile, denoted $F_D$ (as shown in Fig. 2).
2. A Simplified Calculation Method for Hydrodynamic Pressure to Slender Structures

The inertial force is therefore equivalent to the product of the quantity of water and the corresponding acceleration. On the other hand, considering the fluid around the pile, the inertial force at a unit length ($\Delta d$) of pile at depth $z$ is defined as

$$f_I = (C_M - 1) \rho A (\dot{u} - \ddot{x})_z = (C_M - 1) \rho \frac{\pi D^2}{4} (\dot{u} - \ddot{x})_z$$  \hspace{1cm} (1)$$

Then

$$F_I = f_I \Delta d = (C_M - 1) \rho \Delta V (\dot{u} - \ddot{x})$$  \hspace{1cm} (2)$$

where $C_M$ is mass coefficient and also could be called inertial coefficient, $\rho$ is the density of water, and $\Delta V$ is the volume of the underwater pier structure.
2. A Simplified Calculation Method for Hydrodynamic Pressure to Slender Structures

For the fixed rigid pile the drag force is related to the relative velocity of the fluid particles and the piles. The drag force is proportional to the square of velocity of the fluid particles and the projected area of the object in the direction perpendicular to the moving direction of the fluid. As the movement of the fluid is a reciprocating motion, in order to keep the direction, the drag force at a unit height (Δd) of pile at depth z is defined as

$$f_D = C_D \frac{\rho D}{2} (u - \dot{x})_z |u - \dot{x}|_z$$  \hspace{1cm} (3)

Then

$$F_D = f_D \Delta d = C_D \frac{\rho}{2} A_p (u - \dot{x}) |(u - \dot{x})|,$$  \hspace{1cm} (4)

where $A_p$ is the projected area of the unit length of the cylinder in the vertical direction and $C_D$ is the drag force coefficient considering the shape of the pile and the frictional drag force of the column wall.
2. A Simplified Calculation Method for Hydrodynamic Pressure to Slender Structures

Then the hydrodynamic pressure on the surface of the circular cylinder could be defined as

\[ F(x, z, t) = F_I + F_D = (C_M - 1) \rho \Delta V (\mathbf{u} - \mathbf{x}) + \frac{1}{2} C_D \rho A_p (u - \dot{x}) \left| (u - \dot{x}) \right|. \] (5)

As the influence of the structure on the water is ignored, the movement of the structure does not cause a movement of water, so the speed and acceleration of the water particles are zero. The hydrodynamic force on the unit length of the cylinder along the X-axis direction can be expressed as

\[ F_w = -(C_M - 1) \rho \Delta V \ddot{x} - \frac{1}{2} C_D \rho A_p \dot{x} |\dot{x}|. \] (6)
2. A Simplified Calculation Method for Hydrodynamic Pressure to Slender Structures

A dynamic equation of the entire pier system under seismic conditions could be expressed as

$$M\ddot{x} + C\dot{x} + Kx = -M\ddot{x}_g - (C_M - 1)\rho V(\ddot{x} + \ddot{x}_g) - \frac{1}{2}C_D\rho A_p \dot{x} |\dot{x}|.$$ \hspace{1cm} (7)

$$M\ddot{x} + C\dot{x} + Kx = -M\ddot{x}_g - (C_M - 1)\rho V(\ddot{x} + \ddot{x}_g).$$ \hspace{1cm} (8)

The water equivalent added mass on i could be defined as

$$M_{iw} = \sum (C_M - 1)\rho Vl_{ij}.$$ \hspace{1cm} (9)

Therefore, a dynamic equation of the entire pier system under seismic conditions can be expressed as

$$M\ddot{x} + C\dot{x} + Kx = -M\ddot{x}_g - M_w(\ddot{x} + \ddot{x}_g).$$ \hspace{1cm} (10)

FIGURE 3 Dynamic response model for the proposed simplified method
3. The Effect of Hydrodynamic Pressure on the Slender Structure Based on the Simplified Method

For a water-structure interaction simulation analysis, a pile from the high-pile foundation of the main tower of the Nanjing Yangtze River Bridge was selected as a prototype. The bridge is located in a Class II site with eight degrees seismic fortification intensity. The pile is 50 m high, its diameter $D$ is 3 m, the superstructure mass on the pile is $m=2000$ t. In order to make the prototype equivalent to a cylindrical pile, the equivalent consolidation depth method was used to simplify the dynamic effect of soil on the pile. The bottom of the pile is fixed as shown in Fig. 4.

FIGURE 4 Model of cylindrical pile in deep water.
Selected interplate earthquake (Type 1) and intraplate earthquake (Type 2) in Class II sites as far-fault and near-fault seismic waves, included four seismic waves (T1-II-1, T1-II-3, T2-II-1, T2-II-3). Taking the project’s information into account, the proportion method was used to modify the waves and the peak accelerations of 4 seismic waves were adjusted to 0.2 g when the seismic waves were put along the horizontal direction. The four adjusted waves are shown in Fig. 5.
In order to study the effect of hydrodynamic pressure on the internal forces of structures in water subjected to earthquakes, use $R$ to express the influence of the hydrodynamic water on the maximum value of structural dynamic responses:

$$R = \frac{\text{Maximum dynamic response with water} - \text{Maximum dynamic response without water}}{\text{Maximum dynamic response without water}}.$$
3. The Effect of Hydrodynamic Pressure on the Slender Structure Based on the Simplified Method

The cylindrical pile model was studied by using the proposed simplified method. The value of the hydrodynamic inertial force coefficient $C_M$ is chosen as 2.0 in accordance with Code of Hydrology for Sea Harbour. According to the model in Fig. 4, study on the dynamic behavior of the model was made under different seismic waves and from that internal forces of the model were attained. Figures 6 and 7 show how the hydrodynamic pressure influences the shear forces and flexural moments of the pile respectively.

![Graphs showing influence of hydrodynamic pressure on shear forces and flexural moments](image)

FIGURE 6 Influence of hydrodynamic pressure on the shear forces along the cylindrical pile (color figure available online).
3. The Effect of Hydrodynamic Pressure on the Slender Structure Based on the Simplified Method

Figures 7 show how the hydrodynamic pressure influences the shear forces and flexural moments of the pile respectively.

(a) T1-II-1
(b) T1-II-3
(c) T2-II-1
(d) T2-II-3

FIGURE 7 Influence of hydrodynamic pressure on the flexural moments along the cylindrical pile (color figure available online).
3. The Effect of Hydrodynamic Pressure on the Slender Structure Based on the Simplified Method

Study on the influence of hydrodynamic force on the shear force and the flexural moment at the bottom of the cylindrical pile was made, as shown in Fig. 8.

FIGURE 8 Influence of hydrodynamic pressure on the shear force and the flexural moment at the bottom of the cylindrical pile (color figure available online).
3. The Effect of Hydrodynamic Pressure on the Slender Structure Based on the Simplified Method

In Fig. 8, subjected to two types of seismic waves, the bottom shear force and the bottom flexural moment of the cylindrical pile are impacted by different water depths. $R_{\text{shear}}$ and $R_{\text{moment}}$ appear to increase with increasing water depth, $R_{\text{shear}}$ and $R_{\text{moment}}$ under Type 1 seismic waves are larger than them under Type 2 seismic waves. The maximum $R_{\text{shear}}$ reaches 77%, which is at 50 m depth and under T2-II-1 seismic wave, and the maximum $R_{\text{moment}}$ in 40 m depth is only 16.4% when subjected to T2-II-3 seismic wave.
3. The Effect of Hydrodynamic Pressure on the Slender Structure Based on the Simplified Method

The influence of boundary effect on the hydrodynamic pressure

Based on the study of boundary effect on the hydrodynamic pressure of bridge pier, carrying out the regulation study of the variation of the hydrodynamic pressure based on the different depths of water and water models (respectively 10 times, 20 times, 30 times of the short side and infinite boundary waters), establishing the optimal method of the boundary selecting, lay a foundation for the performance design of the long span bridges.

Using the El Centro-wave(NS) as load-waveform, duration of 40s, adjusting the peak acceleration to 400gal according to the site condition.
By the increasing of the water area boundary, the value of the hydrodynamic pressure is decreasing. When the water model is 30 times of the side length of one pier, the maximum of the hydrodynamic pressure is 13159Pa. Comparing with infinite boundary (the maximum value is 13137Pa), the error is 1.3%. Based on this, it can be considered the water model, 30 times of the side length of one pier, basically meets requirements.
3. The Effect of Hydrodynamic Pressure on the Slender Structure Based on the Simplified Method

The comparison of the dynamic characteristics of the cylindrical pile received from the simplified method and the finite element method. The assumption of ideal potential fluid was adopted, so the fluid satisfied the properties such as no heat transfer, viscosity, irrotationality, and imcompressibility, and the fluid boundary also satisfied the small deformation conditions.

The structure model and water model were established, respectively, and were in the coupled state through setting contacts between them. The body of water within a 50 m range from the pile was considered. The calculation model is shown in Fig. 9.

FIGURE 9 Structural model for the finite element method (color figure available online).
4. The Comparison of the Proposed Simplified Method and the Finite Element Method

The natural vibration period of the model was analyzed by using the simplified method and the finite element method; the results are shown in Table 1.

TABLE 1 Comparison of the natural vibration periods obtained from the simplified method and the finite element method

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>The natural vibration period (s)</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>First order</td>
<td>Third order</td>
<td>Deviation rate (%)</td>
<td>Deviation rate (%)</td>
</tr>
<tr>
<td></td>
<td>The simplified method</td>
<td>The finite element method</td>
<td>Deviation rate (%)</td>
<td>The simplified method</td>
</tr>
<tr>
<td>0</td>
<td><strong>5.319</strong></td>
<td>5.347</td>
<td>0.53</td>
<td>0.369</td>
</tr>
<tr>
<td>10</td>
<td><strong>5.319</strong></td>
<td>5.348</td>
<td>0.55</td>
<td>0.370</td>
</tr>
<tr>
<td>20</td>
<td><strong>5.319</strong></td>
<td>5.348</td>
<td>0.55</td>
<td>0.383</td>
</tr>
<tr>
<td>30</td>
<td><strong>5.319</strong></td>
<td>5.360</td>
<td>0.77</td>
<td>0.408</td>
</tr>
<tr>
<td>40</td>
<td><strong>5.338</strong></td>
<td>5.450</td>
<td>2.10</td>
<td>0.427</td>
</tr>
<tr>
<td>50</td>
<td><strong>5.348</strong></td>
<td>5.556</td>
<td>3.89</td>
<td>0.427</td>
</tr>
</tbody>
</table>
4. The Comparison of the Proposed Simplified Method and the Finite Element Method

Table 2 shows the peak displacement and peak acceleration at the top of the cylindrical pile obtained from the two methods.

<table>
<thead>
<tr>
<th>Seismic Wave</th>
<th>Peak displacement at the top of the pile (m)</th>
<th>Peak acceleration at the top of the pile (m/s²)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>The simplified method</td>
<td>The finite element method</td>
</tr>
<tr>
<td>T1-II-1</td>
<td>1.00</td>
<td>1.05</td>
</tr>
<tr>
<td>T1-II-3</td>
<td>1.01</td>
<td>1.06</td>
</tr>
<tr>
<td>T2-II-1</td>
<td>0.33</td>
<td>0.35</td>
</tr>
<tr>
<td>T2-II-3</td>
<td>0.24</td>
<td>0.25</td>
</tr>
</tbody>
</table>
4. The Comparison of the Proposed Simplified Method and the Finite Element Method

Figures 10 display that the displacement-time curves attained from the simplified method and the finite element method coincide with each other, and so do the acceleration-time curves.

(a) T1-III-1
(b) T1-III-3
(c) T2-I-1
(d) T2-III-3

FIGURE 10 Comparison of displacement-time curves of the point at the top of the cylindrical pile obtained from the simplified method and the finite element method (color figure available online).
4. The Comparison of the Proposed Simplified Method and the Finite Element Method

Figures 11 display that the displacement-time curves attained from the simplified method and the finite element method coincide with each other, and so do the acceleration-time curves.

FIGURE 11 Comparison of acceleration-time curves of the point at the top of the cylindrical pile obtained from the simplified method and the finite element method (color figure available online).
5. The Comparison of the Simplified Method and the Shaking Table Experiment

The South Tower of the Nanjing Third Yangtze River Bridge selected for the experiment has 84 m × 29 m elevated pile caps [The Construction Headquarters of Nanjing Third Yangtze River Bridge, 2005]. The layout of the south pier of the Nanjing Third Yangtze River Bridge is shown in Fig. 12. There are 30 piles under the elevated pile caps and there are 15 piles whose diameter is 3 m under each round section. The maximum water depth of the elevated pile caps is 45 m.

FIGURE 12 Layout of the south pier of the Nanjing Third Yangtze River Bridge (unit: m).
5. The Comparison of the Simplified Method and the Shaking Table Experiment

As there are limitations to the testing conditions, only half of the prototype was used in the model. The scale model and the prototype are similar in size and the scales of all physical quantities are shown in Table 3.

<table>
<thead>
<tr>
<th>Physical Quantity</th>
<th>Similarity Coefficient</th>
<th>Physical Quantity</th>
<th>Similarity Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>( S_l = 1.50 )</td>
<td>Inertial Force</td>
<td>( S_F = S_E S_l^2 = 2.75 \times 10^{-3} )</td>
</tr>
<tr>
<td>Modulus of Elasticity</td>
<td>( S_\rho = 6.87 )</td>
<td>Hydrodynamic</td>
<td>( S_P = S_l = 0.02 )</td>
</tr>
<tr>
<td>Equivalent Density</td>
<td>( S_E = 6.87 )</td>
<td>Flexural moment</td>
<td>( S_M = S_E S_l^3 = 5.5 \times 10^{-5} )</td>
</tr>
<tr>
<td>Acceleration</td>
<td>( S_\alpha = 1 )</td>
<td>Frequency</td>
<td>( S_f = S_l^{-1} \sqrt{S_\rho / S_E} = 33.5 )</td>
</tr>
<tr>
<td>Stress</td>
<td>( S_\sigma = S_E = 6.87 )</td>
<td>Time</td>
<td>( S_l = S_l \sqrt{S_E / S_\rho} = 0.03 )</td>
</tr>
<tr>
<td>Strain</td>
<td>( S_\varepsilon = S_\sigma / S_E = 6.87 )</td>
<td>Displacement</td>
<td>( S_u = S_l = 0.02 )</td>
</tr>
</tbody>
</table>

TABLE 3 Similarity coefficient of pier model
5. The Comparison of the Simplified Method and the Shaking Table Experiment

The elevated pile caps are made of reinforced concrete and the piles are made of steel pipes. Wave absorbing devices were installed on the sides of the water tank which are perpendicular to the direction of the input waves in order to eliminate the rebound wave. The arrangement of observation points is shown in Fig. 13.
5. The Comparison of the Simplified Method and the Shaking Table Experiment

The experimental device in air and in water is shown in Fig. 14. In the experiment, the model was subjected to various earthquake waves in air and in water. And the input waves were modified in order to make sure the waves which loaded on the tank were more accurate.

FIGURE 14 Testing Model: (a) pier model in air and (b) pier model in water (color figure available online).
5. The Comparison of the Simplified Method and the Shaking Table Experiment

The experimental model was analyzed by the simplified method proposed in this article; the results of the calculation and the experiment were compared and are shown in Table 4.

TABLE 4 Comparison of natural period and frequency of vibration obtained from the simplified method and the shaking table experiment.

<table>
<thead>
<tr>
<th>Methods</th>
<th>In air</th>
<th>In water</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Frequency(Hz)</td>
<td>Period(s)</td>
</tr>
<tr>
<td>Experimental Value</td>
<td>10.10</td>
<td>0.099</td>
</tr>
<tr>
<td>Calculated Value</td>
<td>9.89</td>
<td>0.101</td>
</tr>
<tr>
<td>R (%)</td>
<td>-2.08</td>
<td>2.13</td>
</tr>
</tbody>
</table>
5. The Comparison of the Simplified Method and the Shaking Table Experiment

The elastoplastic dynamic response time history analysis of the piers in water based on the added mass method

The comparison between the results of experiment and numerical simulation showed that effects of the hydrodynamic pressure to the bridge foundation cannot be ignored based on the added mass method and the shaking table experiment.
The dynamic responses of the model in water subjected to sine waves, the Tianjin wave and Takatori wave, were analyzed by using both the simplified method and the experiment. 1 Hz, 3 Hz, 5 Hz, and 7 Hz sine waves were all taken into account. The results of the peak acceleration value along the model are compared in Figs. 15–17 when the peak acceleration value of the sine waves and seismic waves is 100 Gal or 200 Gal.

FIGURE 15 Distribution of the peak of the horizontal acceleration under harmonic loads (color figure available online).
5. The Comparison of the Simplified Method and the Shaking Table Experiment

According to the comparison, the calculation and experiment results of the dynamic responses of the model in water are almost the same, and there is a certain deviation only in some cases.

FIGURE 16 Distribution of the peak of the horizontal acceleration under the Tianjin Earthquake (color figure available online).
5. The Comparison of the Simplified Method and the Shaking Table Experiment

The maximum deviations under 100 Gal/7 Hz and 200 Gal/7 Hz sine waves are 15.4% and 10.5%, respectively, the maximum deviations under the Tianjin wave and the Takatori wave are 9.0% and 12.5%, respectively. As the simplified method is accurate enough and makes the calculation easier, it is feasible to apply the simplified method in practical structural analysis.

FIGURE 17 Distribution of the peak of the horizontal acceleration under the Takatori Earthquake (color figure available online).
In order to study the effect of the hydrodynamic pressure on the dynamic behavior of the whole bridge structure, based on establishing the finite element computation model of the Ushineohashi Bridge (main span 260m), using the added mass method to make the computation. The results showed that hydrodynamic pressure had large effects on the performance of the bridge.
Application

The Application requirements of study

Taking the performance design of steel arch bridge for example: the structural design of the pier and abutment, including the main beam are closely related to hydrodynamic effect.
6. Conclusions

From all the results shown in this article, it could be concluded as follows.

1. According to Morison’s hydrodynamic theory, the hydrodynamic pressure could be treated as added mass and is composed of inertial force and drag force.

2. The results of the model indicate that water has an obvious effect on the structure. Therefore, it is necessary to study the influence of water on the structures.

3. Comparing the results obtained from these two methods the deviations of the natural vibration periods do not exceed 4%, the displacements and accelerations are almost the same.

4. The simplified method and the experiment led to different results, but the values did not have an obvious deviation. This further demonstrates the validity of the method.
Thank you for your attention!

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谢谢大家！

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