Modal identification of Shanghai World Financial Center both from free and ambient vibration response

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A R T I C L E   I N F O

Article history:
Received 22 July 2010
Revised 17 November 2011
Accepted 18 November 2011

Keywords:
Field tests
Modal identification
High-rise building
Random decrement
Hilbert–Huang transform

A B S T R A C T

The Shanghai World Financial Center (SWFC), a currently built super high-rise building, is located in Lujiazui area. The height of this 101-storey building is 492 m above ground. A set of dynamic field tests were conducted on the building from April to May 2008. To identify the dynamic properties of the building, three output-only modal identification techniques are applied to the ambient and forced vibration measurements. These methods consist of: the Peak-Picking method (PP) combined with the half-power bandwidth method, the Random Decrement based method (RDT) combined with curve-fitting method, and the Hilbert–Huang transform method (HHT). The fundamental frequencies and damping ratios in two translational directions are identified from the free decays of forced vibration tests. The estimated eleven modal frequencies and damping ratios under microtremors from the Peaking-Picking method and the Hilbert–Huang transform method are compared to each other with favorable correlation. The modal frequencies from the finite element analysis and the shaking table test are further studied with the results of field test. The effect of the installed Active Tuned Mass Damper (ATMD) on the damping characteristics of the building is presented. Based on the identified results, accuracy and efficiency of these methods are investigated with the length of Fast-Fourier Transform and the effect of RDT. The modal properties of the SWFC presented in this paper can be used as baseline in future health monitoring.

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1. Introduction

Performance of a super high-rise building subjected to seismic, wind and other dynamic loads depends upon its structural properties such as mass, stiffness, damping and their distribution. Besides the safety during the disasters, the structures should be engineered with sufficient damping to withstand the demand of habitability. Though these structural properties can be determined through the analytical finite element models, the real behaviors of the high-rise building remain to be studied from the full-scale dynamic field test. The modal parameter set can be identified through the dynamic field test, and this valuable information is not just for the analytical model but also for other applications, such as the evaluation of prototypes, model updating, structural health monitoring, and structural control. Various vibration testing techniques are utilized to measure the dynamic properties of the existing structures, which can be divided into two groups. One group named artificial excitation uses mechanical shakers, drop weights, human walking, control devices and so on. The other group named natural excitation uses ambient vibration, wind and earthquake [1].

Recently there are many operational modal identification methods for the measured data analysis including frequency domain, time domain and time-frequency domain approaches. Traditional modal identification methods are developed in frequency domain, such as Peak-Picking (PP) [2], Frequency Domain Decomposition (FDD) [3,4], and Enhanced Frequency Domain Decomposition (EFDD) [5]. The time domain parametric approaches can be divided into Random Decrement based methods (RDT) [6], Natural Excitation Technique (NExT) [7,8], Eigensystem Realization Algorithm (ERA) [8], Autoregressive Moving Average model (ARMA) [9], Bayesian statistical approach [10] and so on. The time-frequency domain approaches are mainly wavelet-type method [11,12] and Hilbert–Huang transform (HHT) method [13,14]. The evaluation of damping has become more important these years due to the higher demand on the precise prediction of the response of high-rise buildings. Traditional damping evaluation methods are referred to half-power bandwidth method in PP [2], logarithmic-decrement method with RDT [6] and Empirical Mode Decomposition-Hilbert Transform (EMD-HT) method [13]. Besides them, there are also some developments in damping estimation these days. A new approach is proposed to identify modal damping ratios from free vibration response [15]. A structural damping identification procedure is presented based on the sensitivity of acceleration response of the model with model updating technique using the
measured acceleration records [16]. The average inverse power ratio method is discussed and proposed for the damping estimation from a frequency response function [17,18].

Detailed system identification tests (for more than ten order modal information) have been successfully applied to a number of bridges such as the Hakuro suspension bridge [8], the Alfred Zampa Memorial Bridge [19], the Gi–Lu bridge [20], and the Qingzhou cable-stayed bridge [21]. Many dynamic field tests on high-rise buildings are performed for the purpose of wind-induced vibration monitoring [22–25] or fundamental modal damping evaluation [26]. These sensor locations, mostly near the roof level among these studies, are not sufficient considering that not all the information of modes is significant in the vibration responses measured from locations near the roof level.

In this study, three representative modal identification techniques were applied to the dynamic field test data collected from the Shanghai World Financial Center (SWFC), a currently built super high-rise building in Shanghai. These methods consist of: (1) Peak-Picking method with half-power bandwidth method in frequency domain; (2) Random Decrement method with logarithmic-decrement method in time domain; and (3) Empirical Mode Decomposition–Hilbert Transform method in time-frequency domain. The measurements on the different floors of the building were analyzed individually by these three methods to identify the modal parameters. There is a discussion about the identified results under ambient vibration between PP method and HHT method. The identified natural frequencies are compared with the analytical finite element model and shaking table test results.

The identified results between PP method and HHT method are shown. The identified natural frequencies are compared with the analytical finite element model and shaking table test results.

The damping characteristics of the building with ATMD action on and off are presented. Recommended lengths of FFT are proposed corresponding to respective selected frequency ranges for the FFT-based methods. Finally, the effect of RDT on the decaying amplitude lines are shown to suggest the importance of RDT in the HHT based method.

2. Field Test of Shanghai World Financial Center

2.1. Introduction of Shanghai World Financial Center

Shanghai World Financial Center (SWFC) is an extraordinary building with a museum at the base, a hotel at the top, and office spaces in between (see Fig. 1). The height of 101-storey building is 492 m above ground. It is located in Lujiazui Financial and Trade district, Shanghai, China. The structure is diagonally symmetrical with a square base plan of 57.95 m × 57.95 m, and the aspect ratio of height to width is 8.49 [27]. Three parallel structural systems including mega-frame structure, reinforced concrete and braced steel services core, and outrigger trusses, are combined to resist vertical and lateral loads, as shown in Fig. 2 [28].

In order to mitigate wind-induced vibration, two identical active tuned mass dampers (ATMDs) are installed on the ninetieth floor (90F), which is illustrated in Fig. 3. The active control feature of the ATMDs is enabled under wind loading. The mass of the damping device is hoisted by the multi-sectional steel cables. The damping devices consist of two parts: multi-section vibration body and drive device. The control force of vibration body is obtained by the feedback motion state variables. These state variables include the acceleration of the floor on which the damping devices are set up, as well as displacement and speed of the vibration body. The active control feature becomes disabled under earthquake excitation and the damping devices perform as passive tuned mass dampers. The devices are locked when the vibration amplitude of vibration body exceeds 110 cm in the passive control state.

2.2. Experimental setup and response data

A set of dynamic field tests were performed on the SWFC from April to May 2008. Fig. 1 also shows the construction stage of SWFC during the dynamic field tests. These tests included ambient vibration tests and forced vibration tests based on the actions of the ATMDs. In order to record the accurate low frequency behaviors of the structure, the acceleration responses at various floors were sampled at a rate of 20 Hz. The resulting Nyquist frequency of
10 Hz was much higher than the frequencies of interest (<2 Hz). A dynamic response range of piezoelectric accelerometers used in this test is from 0.05 Hz to 500 Hz, the measurement capacity range is 0.1 g (1 g = 9.8 m/s²), and the sensibility is 10⁻³ g. Fig. 4 illustrates the typical accelerometers and data acquisition equipment used in this test. During the test, densely distributed accelerometers were placed at different locations in two experimental steps, as shown in Fig. 5. For the first step, the accelerometers were located at tenth floor (10F), fiftieth floor (50F), sixtieth floor (60F), seventieth floor (70F), eightieth floor (80F), and ninetieth floor (90F). Then at the second step, the accelerometers were moved to fifteenth floor (15F), twenty-fifth floor (25F), fifty-fifth floor (55F), sixty-fifth floor (65F), seventy-fifth floor (75F), and eighty-fifth floor (85F). In these steps, both the ambient and forced vibration responses in two translational directions (X, Y) have been recorded. The ambient vibration test data used in this study were collected in 30–40 min segments.

Two types of forced vibration tests were performed on the SWFC by the ATMDs. The first type is that the structure was forced to vibrate by an ATMD in one horizontal direction at an amplitude of 5 cm/s². Then the actuating ATMD was turned off to get the free decay response in this direction. The inherent fundamental damping ratio of the structure with ATMD off is estimated through this forced vibration test. The difference of the second type is that while one actuating ATMD off, the other ATMD was turned on to reduce the vibration in this direction. Figs. 6 and 7 show the acceleration responses of the ninetieth floor (90F) during the two types of forced vibration tests, respectively. The ambient vibrations in the two horizontal directions are also illustrated in Fig. 8.

3. Brief review of modal identification methods used

3.1. Peak-Picking method

The Peak-Picking (PP) method is a traditional frequency domain method using output-only data and already applied to the modal identification of engineering structures. It stems from the fact that the frequency response function (FRF) of a given system will peak at the modal frequencies of the system. With the broadband white noise assumption, the Fourier power spectrum of the response data can be considered as equal to the FRF of the structure at that sensor location. The modal frequencies are identified through the power spectrums from every experimental measurement. The corresponding damping ratios can be estimated with half-power bandwidth method, defined as:

$$\xi_i = \frac{B_i}{2f_i}$$

where $B_i$ is the half-power bandwidth of the spectral peak corresponding to $i$th-order modal frequency $f_i$. 
where \(y(t)\) is a random time history sampled from \(t_i\) to \(t_i + \tau\), \(\tau\) is the equal time length of segments, and \(N\) is the number of segments in the ensemble average.

If a stationary Gaussian random process excites a linear Single-Degree-of-Freedom (SDOF) system, the RD-generated random signature of that response will have similar characteristics as a free vibration response of the linear system under a specified initial condition. The Random Decrement functions are curve-fitted to obtain the logarithmic-decrement functions. Assuming that the logarithmic-decrement function is in the form of Eq. (3), the modal frequencies and damping ratios can be obtained utilizing the linear least-square method:

\[
x(t) = A_0 \cos(\omega_d t) \cdot \exp(-\zeta \omega_d t)
\]

where \(A_0\) is the initial displacement, \(\omega_d\) is the damped natural circular frequency, \(\zeta\) is the damping ratio and \(\omega_0\) is the natural circular frequency.

In this study, the modified RDT method is utilized for analysis, which is to apply the triggering condition in the negative part of the response and then average the extracted time segments for a change in the sign with the extracted positive part of the response time series. The number of the averaged time segments \(N\) is significant increased by the modified RDT method to derive better Random Decrement signature.

### 3.3. Hilbert–Huang transform

The Hilbert–Huang transform method is a two-step data-analysis method [29]. The first step is the empirical mode decomposition (EMD) by which a complicated time history can be turned into a series of intrinsic mode functions (IMFs) that admit well-behaved Hilbert transforms. The second step of the HHT method is implemented by performing the Hilbert transform (HT) to each IMF component [13]. With Hilbert transform, an analytic signal \(z(t)\) for a real-valued function \(y(t)\) can be defined as:

\[
z(t) = y(t) + i\tilde{y}(t) = A(t) \cdot \exp(-i\theta(t))
\]

\[
\tilde{y}(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{y(u)}{t-u} du
\]

where \(\tilde{y}(t)\) is the Hilbert transform of \(y(t)\), \(i\) is the imaginary unit, \(A(t)\) and \(\theta(t)\) are the amplitude and instantaneous phase angle of \(y(t)\), respectively. The instantaneous frequency \(\omega(t)\) is thus the time derivative of \(\theta(t)\). The definition equations of \(A(t), \theta(t), \) and \(\omega(t)\) are shown below:

\[
A(t) = |y(t)^2 + \tilde{y}(t)^2|^{1/2}
\]

\[
\theta(t) = \tan^{-1}\left(\frac{\tilde{y}(t)}{y(t)}\right)
\]

\[
\omega(t) = \frac{d\theta(t)}{dt}
\]

For a linear Single-Degree-of-Freedom (SDOF) system under impulsive loading, the free vibration response function of the system is:

\[
v(t) = A_0 \sin(\omega_d t) \cdot \exp(-\zeta \omega_d t), t \geq 0
\]

where \(\omega_0\) is the natural circular frequency, \(\zeta\) is the damping ratio, \(\omega_d\) is the damped natural circular frequency and \(A_0\) is a constant depending on the intensity of impulsive loading and the mass and frequency of the system. By applying the Hilbert transform method, the signal \(z(t)\) for \(v(t)\) can thus be obtained using Eq. (4):

\[
z(t) = v(t) + i\tilde{v}(t) = A(t) \cdot \exp(-i\theta(t))
\]

For a special case in which \(\zeta\) is small and \(\omega_d\) is large, the amplitude \(A(t)\) and the phase angle \(\theta(t)\) for the SDOF system can be obtained as follows [30]:

![Fig. 5. The elevation of Shanghai World Financial Center and the accelerometer distribution of the field test.](image)
Fig. 6. Acceleration response measured on the ninetieth floor in two horizontal directions during the first type of forced vibration test (ATMD off for the free decays): (a) X direction; (b) Y direction.

Fig. 7. Acceleration response measured on the ninetieth floor in two horizontal directions during the second type of forced vibration test (ATMD on for the free decays): (a) X direction; (b) Y direction.

Fig. 8. Ambient vibration measurement on the ninetieth floor in two horizontal directions: (a) X direction; (b) Y direction.
A(t) = A_0 \cdot \exp\left(-\frac{\zeta \omega_0 t}{2}\right) \\
\theta(t) = \omega_d t - \frac{\pi}{2}

By introducing the logarithmic and differential operators to Eqs. (11) and (12), respectively, one obtains:

\[
\ln A(t) = -\frac{\zeta \omega_0 t}{2} + \ln A_0
\]

\[
\omega(t) = \frac{d\theta(t)}{dt} = \omega_d
\]

Therefore, the damped natural circular frequency \( \omega_d \) can be identified from instantaneous frequency \( \omega(t) \) by Eq. (14). With the identified \( \omega_d \) and the slope \( -\zeta \omega_0 \) of the straight line of the decaying

---

### Table 1

Identified natural frequencies and damping ratios of the SWFC based on free decays with ATMD off.

<table>
<thead>
<tr>
<th>Method</th>
<th>X</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>RDT-curve fitting method</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Value</td>
<td>0.1562</td>
<td>0.1567</td>
</tr>
<tr>
<td>Damping ratio</td>
<td>0.00509</td>
<td>0.00423</td>
</tr>
</tbody>
</table>

| Hilbert–Huang transform method |       |       |
| Mode information              |       |       |
| Value                       | 0.1563| 0.1570|
| Damping ratio               | 0.00525| 0.00439|

---

Fig. 9. Instantaneous functions of the first modal response by Hilbert transform method: (a) phase angle and linear least-squares fit; (b) amplitude and linear least-squares fit.

Fig. 10. Instantaneous functions of the second modal response by Hilbert transform method: (a) phase angle and linear least-squares fit; (b) amplitude and linear least-squares fit.
amplitude $A(t)$ in a semi-logarithmic scale, the damping ratio $\zeta$ can be identified from the function $\omega_d = \omega_0(1 - \zeta^2)^{1/2}$. It should be noted that the Random Decrement technique is performed on the target IMF to derive the free vibration response $v(t)$ before the Hilbert transform.

4. Modal identification results

This section presents the identified natural frequencies and damping ratios both from free decays and ambient vibration measurements. In purpose of comparison of the results from the free decay and ambient vibration, the free and ambient vibration data used in this section were all measured with the ATMD action off, for example as shown in Figs. 6 and 8.

4.1. Results based on free vibration data

The modal identification techniques, RDT-curve fitting and Hilbert–Huang transform method Eq. (9)–(14), were applied to estimate the fundamental modal parameters based on free vibration test data. Due to the fact that the free vibration data are of shorter duration than the ambient data, the Peak-Picking method is not preferred here. The estimated fundamental frequencies and damping ratios in the two translational directions are listed in Table 1. The instantaneous functions of the first two modal responses derived from the Hilbert transform are illustrated in Figs. 9 and 10. The damping ratios identified from the RDT-curve fitting method are further considered as reference values to evaluate the accuracy of the results from ambient vibration in the following section. It is noted that the fundamental frequencies in the two directions are close to each other, and the identified damping ratios of the structure are around 0.5%. The low damping characteristics of the first two modes partially verify the necessity of the installation of the ATMDs on the high-rise building.

4.2. Results based on ambient vibration data

According to the practical engineering experience, accurate estimation values of mode frequencies and damping ratios cannot be obtained through the PP method only by limited test locations. Fig. 11 shows the power spectrums in two translational directions from different test locations, which are fifteenth floor, sixty-fifth floor, and eightieth floor. It is understood that some higher mode frequency peaks may be not significant in the power spectrum.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Data location</th>
<th>Pick-Peaking method</th>
<th>Frequency (Hz)</th>
<th>Damping ratio</th>
<th>Hilbert–Huang transform method</th>
<th>Frequency (Hz)</th>
<th>Damping ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>90F</td>
<td>0.1550</td>
<td>0.0059</td>
<td></td>
<td>0.1554</td>
<td>0.0063</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>90F</td>
<td>0.1575</td>
<td>0.0046</td>
<td></td>
<td>0.1579</td>
<td>0.0048</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>85F</td>
<td>0.4858</td>
<td>0.0066</td>
<td></td>
<td>0.4878</td>
<td>0.0049</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>15F</td>
<td>0.5737</td>
<td>0.0074</td>
<td></td>
<td>0.5496</td>
<td>0.0070</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>25F</td>
<td>0.865</td>
<td>0.0073</td>
<td></td>
<td>0.5730</td>
<td>0.0069</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>65F</td>
<td>0.9375</td>
<td>0.0050</td>
<td></td>
<td>0.9419</td>
<td>0.0049</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>55F</td>
<td>0.9717</td>
<td>0.0066</td>
<td></td>
<td>0.9747</td>
<td>0.0054</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>80F</td>
<td>1.0254</td>
<td>0.0085</td>
<td></td>
<td>1.0270</td>
<td>0.0032</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>25F</td>
<td>1.3965</td>
<td>0.0124</td>
<td></td>
<td>1.3872</td>
<td>0.0102</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>65F</td>
<td>1.4697</td>
<td>0.0078</td>
<td></td>
<td>1.4665</td>
<td>0.0107</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>85F</td>
<td>1.5283</td>
<td>0.0054</td>
<td></td>
<td>1.5279</td>
<td>0.0053</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 11. Power spectrums in two translational directions from the response to ambient excitations: (a) X direction on the eightieth floor; (b) X direction on the sixty-fifth floor; (c) X direction on the fifteenth floor; (d) Y direction on the eightieth floor; (e) Y direction on the sixty-fifth floor; (f) Y direction on the fifteenth floor.
Fig. 12. The original ambient measurement on the ninetieth floor and its IMF components by EMD method.
from the locations near the anti-node of the mode. For example, the peak corresponding to 1.4697 Hz is significant in Fig. 11b but not apparent in Fig. 11a. Additionally, the fundamental frequency of 0.1575 Hz in the Y direction is not as clearly shown in Fig. 11e as in Fig. 11d and f. By analysis of all the significant peaks of the power spectrums from the twelve floors, the first eleven modal frequencies are identified. The corresponding damping ratios are obtained by applying the half-power bandwidth method. The relative levels and the curve shapes of the modal frequency peaks in the power spectrum are the two main reasons to select the data among the twelve floors for damping evaluation. The relatively higher and steeper peak is preferred which indicates more prominent vibration component in the measurement and less energy leak of this mode in the frequency domain. Different lengths of FFT were utilized ranging from 2048 to 16384 for the identification of the eleven modes. Both the natural frequencies and damping ratios are listed in Table 2 with the corresponding location information of the used measurement data.

The HHT method is applied to each selected time series of structural acceleration response to obtain the Hilbert spectrum in the time-frequency domain. The first step is empirical mode decomposition to obtain the intrinsic mode functions (IMFs). A series of IMFs obtained from the measured data in the X direction at the ninetieth floor by empirical mode decomposition are plotted in Fig. 12. Since there are still some random components in the target IMF that would affect the accuracy of the identification, the RDT is then applied to the obtained IMF to extract the free modal responses, which is then analyzed by the HT method [13,14,31]. The triggering level was chosen as 1.0 $\cdot \sigma$, where $\sigma$ is the root-mean-square of the time history. The optimal duration of segment used in this study ranged from 40 s (eleventh-order mode) to 200 s (first-order mode) in the purpose that the RDT function is fully decayed and the vibration is not becoming too small. Fig. 13 shows the first and eleventh-order free modal response time series resulting from the seventh IMF of the measured data at the ninetieth floor in the X direction and the third IMF of the measured data at
the eighty-fifth floor in the X direction, respectively. The last step is to apply the Hilbert transform to every free modal response time series to yield the instantaneous phase angle and amplitude functions. Both the instantaneous phase angle and amplitude functions from the first and eleventh-order free modal response time series are shown in Figs. 14 and 15 with their linear least-squares fits. The identified modal frequencies and damping ratios of SWFC by the HHT method are also listed in Table 2.

The identified damping ratios and the rates of the identified values to reference values.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Reference value (free decay)</th>
<th>PP method</th>
<th>HHT method</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Damping ratio</td>
<td>Damping ratio</td>
<td>Rate</td>
</tr>
<tr>
<td>1</td>
<td>0.00509</td>
<td>0.0059</td>
<td>1.159</td>
</tr>
<tr>
<td>2</td>
<td>0.00423</td>
<td>0.0046</td>
<td>1.087</td>
</tr>
</tbody>
</table>

4.3. Comparison

Fig. 16 plots the identified natural frequencies and corresponding damping ratios by Peak-Picking method and the Hilbert–Huang Transform method. It is seen from Table 2 and Fig. 16 that the estimated first eleven modal frequencies are approximately identical to each other between these two methods. Although the identified damping ratios do not show the same agreement as the modal
frequencies, the results are still applicable in practical engineering, when considering the inherent uncertainty in damping prediction and estimation [6]. It is suggested that the accuracy of 5–10% would be appropriate for damping evaluation based on steady-state forced vibration testing [32]. In this paper, the reference values were assumed to be the damping ratios estimated from free decays by curve-fitting method. The rates of damping ratio between the values identified from ambient vibration to the reference values are listed in Table 3. It is seen that the rates of different method in the $X$ direction are 1.159 and 1.238, while 1.087 and 1.135 in the $Y$ direction. The accuracy between the free decays and the ambient vibration is acceptable in damping evaluation.

Besides the field measurement, a three-dimensional numerical model of the SWFC was developed in the finite element analysis software ANSYS. The first five natural frequencies and mode shapes obtained by the analytical finite element model are illustrated in Fig. 17. It shows that the first three modes of vibration are the lateral bending in the $X$ direction, the lateral bending in the $Y$ direction, and the torsion mode. According to the designed similarity ratio, the modal frequencies of the prototype can also be calculated from the identified modal frequencies of the experimental structure of the shaking table test. The first three modal frequencies of the building identified from the field test by PP method are compared with their analytical counterparts obtained from the finite element model and the shaking table test, which is listed in Table 4. It is noted that the identified modal frequencies have similar values when compared between these research areas.

### Table 4

<table>
<thead>
<tr>
<th>Mode</th>
<th>Field dynamic test Frequency (Hz)</th>
<th>Finite element analysis Frequency (Hz)</th>
<th>Shaking table test Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.1550</td>
<td>0.1561</td>
<td>0.176</td>
</tr>
<tr>
<td>2</td>
<td>0.1575</td>
<td>0.1837</td>
<td>0.176</td>
</tr>
<tr>
<td>3</td>
<td>0.4858</td>
<td>0.3598</td>
<td>0.411</td>
</tr>
</tbody>
</table>

Note: The structure used in the finite element model is with ATMD and the experimental structure used in the shaking table test is without ATMD.

### Table 5

<table>
<thead>
<tr>
<th>Direction</th>
<th>Frequency (Hz)</th>
<th>ATMD action</th>
<th>Damping rate (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X$</td>
<td>0.1562</td>
<td>Off</td>
<td>0.509</td>
</tr>
<tr>
<td></td>
<td></td>
<td>On</td>
<td>3.404</td>
</tr>
<tr>
<td>$Y$</td>
<td>0.1567</td>
<td>Off</td>
<td>0.423</td>
</tr>
<tr>
<td></td>
<td></td>
<td>On</td>
<td>3.865</td>
</tr>
</tbody>
</table>

Besides the modal identification of the structure under micro tremors, the effect of the installed ATMDs on the damping properties were also evaluated through the analysis on the two types of the forced vibration tests. The measurements are shown above in Figs. 6 and 7. The intuitive difference between these two figures is the significant different duration time of free decays to low vibration levels (<0.01 m/s²), which is nearly 400 and 50 s in Figs. 6 and 7, respectively. The comparison of the identified damping ratios by the curve-fitting method on the free decays are listed in Table 5. It is noted that the fundamental damping in the two translational directions are increased approximately seven-fold and nine-fold, respectively, when the ATMDs are turned on.

There is a general accepted opinion that the FFT-based method seems to overestimate the damping ratio compared to other methods because of the request for the length and stationary of data. There must be at least four spectral lines in the half-power bandwidth of each mode for limit bias errors [32]. In practical modal identification, it is not convenient to check the contained spectral line number. As a alternative, the length of FFT is proposed here to study the accuracy. A convergence study of the effect of the frequency resolution $D_f$ on the damping ratio identification was performed as illustrated in Fig. 18. The dashed lines show the damping estimations from the free decays with ATMD action off, as reference values acting in the two horizontal directions. It is seen that the estimations of damping shown by solid line converge to the reference value while the frequency resolution decreases from 0.0195 Hz to 0.0012 Hz. Note that the frequency resolution value 0.0195 Hz corresponds to the length of FFT of 1024, while 0.0012 Hz relates to 16384. The frequency resolution here is directly determined by the length of FFT. Depending on the identified results from field test, a series of recommended length of FFT to utilize is proposed to meet the requirement of frequency resolution.
resolution. If the interested modal frequency is in the range of \( \frac{1}{2^{n+1}} \sim \frac{f_{\text{nst}}}{2^n} \), the recommended length of FFT should be \( 1024 / 2^n \), where \( f_{\text{nst}} \) is the Nyquist frequency of the test.

The Hilbert–Huang transform method is utilized to analyze nonlinear and nonstationary time series in the frequency-time domain. If the HHT method is applied to the stationary ambient vibration measured data, the Random Decrement technique is necessary to obtain the free vibration modal response. Two comparative cases were performed. One case used the RDT method on the target IMF before the Hilbert transform. The other case performed the Hilbert transform directly on the target IMF without RDT. Two decaying amplitude lines from both cases are plotted in Fig. 19. The decaying amplitude line, derived without RDT as shown in Fig. 19b, becomes unstable with large amplitude oscillation, which makes it unsuitable for use of linear least-squares fit technique for damping estimation. It illustrates that the free vibration modal response from the Random Decrement technique ensures the accuracy of the Hilbert transform for modal identification of structural systems.

### 6. Conclusions and remarks

This study has presented a data analysis with three modal identification procedures to obtain dynamic properties of the Shanghai World Financial Center both from the free decays and ambient vibration response. Three representative modal identification techniques are applied to the measured vibration data from a set of dynamic field tests. These tests provided a unique opportunity to obtain the modal information of the super high-rise building from
artificial excitation and natural excitation, respectively. The long-period and inherent low-damping characteristics of the super high-rise building are identified from the field test. Because of the twelve test locations distributed along the elevation in different floors, the identified natural frequencies and damping ratios are given out with good confidence. The accuracy of the methods has been further investigated. There are some conclusions drawn from the study: (1) the identified natural frequencies from the different methods in different domains are in excellent agreement for the eleven modes; (2) the discrepancy of the estimated damping ratios under ambient vibration is relatively larger than that of the natural frequencies, but still shows promise for use in practical engineering; (3) the optimal test locations for different modes are distributed from twenty-fifth floor (25F) to ninetieth floor (90F); (4) the identified natural frequencies of the prototype agree with the results of finite element model and shaking table test; (5) the fundamental damping ratios in two directions have been increased significantly with the actions of the ATMDs; (6) a series of recommended length of FFT is proposed to meet the requirement of frequency resolution in practice; and (7) the RDT should be applied between the EMD and HT method when facing the ambient vibration response records.

The modal identification results from this study provide the eleven modal properties of the Shanghai World Financial Center, which can be in the application of the structural health monitoring and structural damage detection to the super high-rise building under ambient vibration.

Acknowledgements

The authors are thankful for the assistance provided by Dr. Xianqun Guo during the field test on the Shanghai World Financial Center.

References